#### **STEADY UNIFORM FLOWS**

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References

Uniform flows are open-channel flows, which do not change with distance at a particular time. The flow condition can be obtained via momentum concept. In this chapter, qualifications and the momentum concept for the uniform flow are given. Then, three widely-used formulas are introduced. The origin of these formulas are traced mechanistically and related coefficients are discussed.

# **1. Qualifications for Uniform Flow**

The uniform flow has the following features:

(1) the depth, water area, velocity, and discharge at every section of the channel reach are constant.

(2) the slope of energy line  $(S_e)$ , the slope of water surface  $(S_w)$ , and the slope of the channel bottom  $(S<sub>o</sub>)$  are the same.

A constant velocity may be interpreted as a constant time-averaged velocity, i.e., This should mean that the flow possesses a constant velocity at every point on the channel section within the channel reach. That is, the velocity distribution across the channel section is unaltered in the reach. Such a stable pattern is attained when the boundary layer is fully developed. Uniform flow is considered to be steady only because unsteady uniform flow is practically nonexistent. tiant velocity may be interpreted as a constant time-averaged velocity, i.e., This should<br>hat the flow possesses a constant velocity at every point on the channel section within<br>nmel reach. That is, the velocity distribut

#### **2. Shear Stress in Uniform Flow**

A uniform flow is developed if the gravity force is balanced by the resistance. Consider the force balance in the uniform flow (momentum approach). The gravity force acting on the fluid element is given by *Final islands in the stable pattern is attained when the boundary layer is fully developed. Uniform*<br> **For Postands at able pattern is attained when the boundary layer is fully developed. Uniform<br>
<b>For Pick** on the stabl

$$
F_{\varphi} = \gamma A dx \sin \theta \tag{1}
$$

The resisting force due to shear along the wetted perimeter is

$$
F_f = \tau_0 P dx \tag{2}
$$

Equating Eq.(1) and Eq.(2) results in

$$
\tau_0 = \gamma R_h S_0 \tag{3}
$$

ar Stress in Uniform Flow<br>
form flow is developed if the gravity force is balanced by the resistance. Consider the<br>
alance in the uniform flow (momentum approach). The gravity force acting on the fluid<br>
at is given by<br>  $F$ **2. Shear Stress in Uniform Flow**<br>
A uniform flow is developed if the gravity force is balanced by the resistance. Consider the<br>
force balance in the uniform flow (momentum approach). The gravity force acting on the fluid flows and Eq.(3) is valid for any channels of arbitrary cross sections. From Eq.(3), the friction slope is defined by

Steady Uniform Flow

$$
S_f = \frac{\tau_0}{\gamma R_h}
$$

which is not the same as  $S_0$  ( $S_w$  or  $S_e$ ) for flows other than the uniform flow. The friction slope means that the slope is obtained from the momentum concept.



Figure 1. Force balance of the flow in a prismatic open channel

### **3. Uniform Flow Formulas**

### **3.1 Chezy Formula (1769)**

The history of uniform flow begins with Antoine de Chezy with data on a canal upstream of Paris. The Chezy's formula takes the form of

$$
V = C \sqrt{R_h S_0} \tag{4}
$$

1. Force balance of the flow in a prismatic open channel bed<br> **Example 11.** Force balance of the flow in a prismatic open channel<br> **form Flow Formulas**<br> **Example 1769)**<br> **Example 18.** The Chezy's formula takes the form of where  $V =$  mean velocity,  $R_h$  = hydraulic radius,  $S_0 =$  bed slope, and  $C =$  Chezy coefficient representing the wall roughness. It is seen that *C* is of  $[L^{1/2}/T]$ .

Chezy's formula can be derived theoretically. In fluid mechanics, the shear stress at the bottom is represented by

$$
\tau_0 = c_f \rho \frac{V^2}{2}
$$

Steady Uniform Flow<br>
<sup>2</sup>'s formula can be derived theoretically. In fluid mechanics, the shear stress at the bottom<br>
csented by<br>  $\tau_0 = c_f \rho \frac{V^2}{2}$ <br>  $c_f$  is a geometric factor influenced by the boundary roughness (or fl where  $c_f$  is a geometric factor influenced by the boundary roughness (or flow resistance coefficient). That is, the resisting force is obtained by assuming that the force per unit stream bed area is proportional to the squared mean velocity. Then, the total resisting force is Steady Uniform Flow<br>
<sup>2</sup>S formula can be derived theoretically. In fluid mechanics, the shear stress at the bottom<br>
<sup>76</sup><sub>6</sub> =  $c_f \rho \frac{V^2}{2}$ <br>  $c_f$  is a geometric factor influenced by the boundary roughness (or flow resis

$$
F_f = KV^2Pdx
$$
\n(5)

Steady Uniform Flow<br>
is represented by<br>
is represented by<br>  $\tau_0 = c_f \rho \frac{V^2}{2}$ <br>
where  $c_f$  is a geometric factor influenced by the boundary roughness (or flow resistance<br>
coefficient). That is, the resisting force is obt where K is a proportionality. From Eqs.(1) and (5), it is obtained that  $V = C \sqrt{R_h S_0}$  with *Chezy's* formula can be derived theoretically. In fluid mechanics, the shear stress at the bottom<br>is represented by<br> $\tau_0 = c_f \rho \frac{V^2}{2}$ <br>where  $c_f$  is a geometric factor influenced by the boundary roughness (or flow resi based on the momentum concept. coefficient). That is, the resisting force is obtained by assuming that the force per unit stream<br>bed area is proportional to the squared mean velocity. Then, the total resisting force is<br> $F_y = KV^2 P dx$  (5)<br>where K is a prop *F<sub>f</sub>* = *KV*<sup>2</sup>*Pdx* (5)<br> *K* is a proportionality. From Eqs.(1) and (5), it is obtained that  $V = C\sqrt{R_s S_0}$  with<br>  $\frac{\gamma}{K}$ . Therefore, it can clearly be understood that the Chezy's formula was proposed<br>
on the momentum

### **3.2 Manning Formula (1889)**

For incompressible, steady flows at a constant depth (uniform flow) in a prismatic open- Manning's formula such as **g Formula (1889)**<br>
ressible, steady flows at a constant depth (uniform fl<br>
Manning formula is widely used. Substituting  $C = C_m h$ <br>
ormula such as<br>  $\int_{\frac{m}{2}} R_h^{2/3} S^{1/2}$ <br>
lue of  $C_m$  is 1 and 1.49 for SI and USC units,

$$
V = \frac{C_m}{n} R_h^{2/3} S^{1/2} \tag{6}
$$

where the value of  $C_m$  is 1 and 1.49 for SI and USC units, respectively. This happens because Manning's equation is dimensionally non-homogeneous. The following Table delivers representative Manning's roughness coefficients for various boundary materials.

Actually, Philippe Gauckler proposed the same type of formula very similar to Eq.(6) three years earlier (Hager, 2015). In order to take Gauckler's contribution into account, Eq.(6) is also called Gauckler-Manning formula. Later, Strickler proposed the following formula: Steady Uniform Flow<br>
Philippe Gauckler proposed the same type of formula very similar to Eq.(6) three<br>
Fr (Hager, 2015). In order to take Gauckler's contribution into account, Eq.(6) is also<br>
ckler-Manning formula. Later, Steady Uniform Flow<br> *Vg*, Philippe Gauckler proposed the same type of formula very similar to Eq.(6) three<br> *Variance Cauckler-Manning formula.* Later, Strickler proposed the following formula:<br> *V* = 21.1(2gS<sub>0</sub> $R_h$ )<sup>1/</sup>

$$
V = 21.1 \left( 2gS_0 R_h \right)^{1/2} \left( \frac{R_h}{d_m} \right)^{1/6} \tag{7}
$$

Steady Uniform Flow<br>
Steady Uniform Flow<br>
Philippe Gauckler proposed the same type of formula very similar to Eq.(6) three<br>
lier (Hager, 2015). In order to take Gauckler's contribution into account, Eq.(6) is also<br>
auckle where  $d_m$  is the median sediment size representing the size of equivalent spheres, of which the surface of the channel is composed. It is interesting to note that Eq.(7), Manning-Strickler's formula, is dimensionally homogeneous.

(Q) What is the dimension of *n* and  $C_m$ ?

Table 1. Manning's Roughness Coefficient (Chow, 1959)

Material		<b>Typical</b> Manning roughness coefficient
Concrete	- -	0.012
Gravel bottom with sides - concrete	mortared stone $-$ riprap	0.020 0.023 0.033
Natural stream channels Clean, straight stream Clean, winding stream Winding with weeds and pools With heavy brush and timber		0.030 0.040 0.050 0.100
Flood Plains Pasture Field crops Light brush and weeds Dense brush Dense trees		0.035 0.040 0.050 0.070 0.100

#### **3.3 Darcy-Weisbach Formula**

3.3 Darcy-Weisbach Formula<br>
If Chezy coefficient (*C*) is replaced by  $\sqrt{8g/f}$ , then the following Darcy-Weisbach formula<br>
is obtained:<br>  $V = \sqrt{\frac{8g}{f}} \sqrt{R_h S}$  (7) is obtained: **g Steady Uniform Flow**<br> **g zy coefficient** (*C*) is replaced by  $\sqrt{8g/f}$ , then the following Darcy-Weisbach formula<br>
ined:<br>  $V = \sqrt{\frac{8g}{f}} \sqrt{R_h S}$  (7)<br>
the roughness coefficient *f* is given by<br>  $f = f n(k/R_h, Re)$  (8) Steady Uniform Flow<br>
eisbach Formula<br>
Ticient (C) is replaced by  $\sqrt{8g/f}$ , then the following Darcy-Weisbach formula<br>  $\frac{\sqrt{g}}{\sqrt{g}}\sqrt{R_s S}$  (7)<br>
ghness coefficient f is given by<br>  $(k/R_s, Re)$  (8)<br>
the roughness height. Value *f fn k R* <sup>=</sup> (8) *f* exceptional Formula<br> *f* y coefficient (*C*) is replaced by  $\sqrt{8g/f}$ , then the following Darcy-Weisbach formula<br>
red:<br>  $f' = \sqrt{\frac{8g}{f}} \sqrt{R_a S}$  (7)<br> *f* =  $\sqrt{n} (k/R_a, Re)$  (8)<br> *f* =  $\int n(k/R_a, Re)$  (8)<br> *f* =  $\int n(k/R_a, Re)$  ( *f* =  $\sqrt{\frac{8g}{f}}$  =  $\sqrt{\frac{8g}{f}}$  =  $\sqrt{\frac{8g}{f}}$  =  $\sqrt{R_s S}$  (7)<br> *f* =  $\sqrt{\frac{8g}{f}}$   $\sqrt{R_s S}$  (7)<br> *f* =  $\frac{1}{\sqrt{f}}$   $\sqrt{R_s R_s}$  (8)<br> *f* =  $\int n(k/R_s, Re)$  (8)<br> *f* =  $\int n(k/R_s, Re)$  (8)<br> *f* =  $\int n(k/R_s, Re)$  (8)<br> *f* =  $\int n(k/R_s, Re)$ 

$$
V = \sqrt{\frac{8g}{f}} \sqrt{R_h S} \tag{7}
$$

where the roughness coefficient *f* is given by

$$
f = fn(k/R_h, \text{Re})\tag{8}
$$

in which  $k$  is the roughness height. Values of roughness coefficient  $f$  are given in the Moody diagram which is obtained from experiments of pipe flows. The expressions for *f* are is obtained:<br>  $V = \sqrt{\frac{8g}{f}} \sqrt{R_s S}$  (7)<br>
where the roughness coefficient f is given by<br>  $f = f n(k/R_s, Re)$  (8)<br>
in which k is the roughness height. Values of roughness coefficient f are given in the Moody<br>
diagram which is obta

$$
f = \frac{24}{\text{Re}} \tag{9a}
$$

$$
f = \frac{0.223}{\text{Re}^{1/4}}
$$
 500 < Re \le 25,000 (9b)

$$
\frac{1}{\sqrt{f}} = 2\log \text{Re}\sqrt{f} + 0.4
$$
 Re > 25,000 (9c)

(7)<br>
ghness coefficient *f* is given by<br>  $k/R_n$ , Re) (8)<br>
the roughness height. Values of roughness coefficient *f* are given in the Moody<br>
is obtained from experiments of <u>pipe flows</u>. The expressions for *f* are<br>
Re  $\leq$ (7)<br>
ughness coefficient f is given by<br>  $h(k / R_n, \text{Re})$  (8)<br>
the roughness height. Values of roughness coefficient f are given in the Moody<br>
ch is obtained from experiments of <u>pipe flows</u>. The expressions for f are<br>  $\frac{4}{$ where the roughness coefficient f is given by<br>  $f = f n(k / R_s, \text{Re})$ <br>
in which *k* is the roughness height. Values of roughness coefficient f are given in the Moody<br>
diagram which is obtained from experiments of <u>pipe flows</u>. or<br>
in which *k* is the roughness height. Values of roughness coefficient *f* are given in the Moody<br>
diagram which is obtained from experiments of <u>pipe flows</u>. The expressions for *f* are<br>  $f = \frac{24}{Re}$  Re  $\leq 500$  (9a)

$$
\frac{1}{\sqrt{f}} = 2\log\frac{R_h}{k} + 2.16
$$
 Re > 25,000 (9d)

is obtained from experiments of <u>pipe flows</u>. The expressions for f are<br>  $Re \le 500$  (9a)<br>
23<br>
23<br>
23<br>
23<br>
24<br>
24<br>
24<br>
260 < Re  $\le 25,000$  (9b)<br>
21<br>
29  $Re \sqrt{f} + 0.4$   $Re > 25,000$  (9c)<br>
21<br>
21<br>
22  $Re \sqrt{f} + 0.4$   $Re > 25,000$  ( ch is obtained from experiments of <u>pipe flows</u>. The expressions for f are<br>  $R \le 500$  (9a)<br>  $\frac{223}{16e^{1/4}}$  500 < Re  $\le 25,000$  (9b)<br>
veloped turbulent flows over hydraulically-smooth boundary with  $R \ge 25,000$ <br>  $= 2 \log$ where *k* is the equivalent size of the Nikuradse type surface roughness and  $u_*$  is the shear  $f = \frac{0.223}{Re^{1/4}}$  500 < Re ≤ 25,000 (9b)<br>
For fully-developed turbulent flows over hydraulically-smooth boundary with Re > 25,000<br>  $\frac{1}{\sqrt{f}} = 2 \log Re \sqrt{f} + 0.4$  Re > 25,000 (9c)<br>
and for fully-developed turbulent flows o turbulent flows over hydraulically rough surface.

Among three resistance factors, the Darcy-Weisbach *f* has the best theoretical background. It is non-dimensional, and its values for steady uniform flows are given in the Moody diagram. However, it should be emphasized that the roughness coefficient *f* in Darcy-Weisbach formula is a local quantity as indicated by the above relationship whereas the roughness coefficients in the other formulas are reach-averaged quantities. The reason why Darcy-Weisbach formula is not so popular in the practical hydraulics is that the roughness coefficient comes from the pipe flow experiments. That is, there is no Moody diagram for the open-channel flow, and  $f - Re$ relationship changes according to channel geometry. formulas are reach-averaged quantities. The reason why Darey-Weisbach formula is<br>ppluar in the practical hydraulics is that the roughness coefficient comes from the pipe<br>eriments. That is, there is no Moody diagram for th formulas are reach-averaged quantities. The reason why Darcy-Weisbach formula is<br>pular in the practical hydraulics is that the roughness coefficient comes from the pipe<br>eriments. That is, there is no Moody diagram for the

#### **3.4 Dimensional Consideration**

Manning's, Chezy's, and Darcy-Weisbach's formulas were originally developed empirically although theoretical numerous attempts were made later. From the three relationships, it is clear that

$$
\sqrt{\frac{8}{f}} = \frac{C}{\sqrt{g}} = \frac{C_m}{\sqrt{g}} \frac{R_h^{1/6}}{n}
$$
\n(10)

The above equation reveals that

(a) The Chezy *C* has the dimension of  $\sqrt{g}$ .

(b)  $C_m$  in the Manning's formula has the dimension of  $\sqrt{g}$  because it is unreasonable to assume *n* changes with changing *g*. Therefore, the Manning's *n* has the dimension of  $[L^{1/6}]$ .

Although *n* has a dimension of  $[L^{1/6}]$ , in practice the same numerical value of *n* is used in English system as in SI system, and hence the constant 1.49 absorbs not only the dimension of *g* but also the conversion factor from SI system. Steady Uniform Flow<br> *n* has a dimension of [L<sup>1/6</sup>], in practice the same numerical value of *n* is used in<br>
ystem as in SI system, and hence the constant 1.49 absorbs not only the dimension of<br>
the conversion factor fro Steady Uniform Flow<br> *f* a dimension of [L<sup>1/8</sup>], in practice the same numerical value of *n* is used in<br>
as in SI system, and hence the constant 1.49 absorbs not only the dimension of<br> *f* conversion factor from SI sys

# **3.5 Variation of** *n* **with Surface Roughness**

It is a well known fact that for a given rough surface the value of *n* hardly changes with the depth or discharge of the flow provided the roughness elements of the channel are statistically homogeneous and randomly distributed. It is clear that

$$
\frac{n}{k^{1/6}} = f(R_h / k, \text{Re})
$$
\n(11)

As can be seen in Eq.(9d), only for fully-developed turbulent flows over a rough boundary, English system as in SI system, and hence the constant 1.49 absorbs not only the dimension of<br>
2.5 Variation of *n* with Surface Roughness<br>
3.5 Variation of *n* with Surface Roughness<br>
1. is a well known fact that for a g  $n/k^{1/6}$  is a function of  $R_h/k$  but not Re. If *n* is truly a measure of the surface roughness, **3.5 Variation of** *n* **with Surface Roughness**<br> **13.5 Variation of** *n* **with Surface Roughness<br>
it is a well known fact that for a given rough surface the value of** *n* **hardly changes with the<br>
depth or discharge of the flow**  $n/k^{1/6}$  will be a constant independent of either  $R_h/k$  or Re. Strickler (1923) proposed that ncous and randomly distributed. It is clear that<br>  $\frac{n}{k^{1/6}} = f(R_s / k, Re)$  (11)<br>
be seen in Eq.(9d), only for fully-developed turbulent flows over a rough boundary,<br>
is a function of  $R_s / k$  but not  $Re$ . If *n* is truly a mea *n C k* is and randomly distributed. It is clear that<br>  $f(R_k/k, Re)$  (11)<br>
cen in Eq.(9d), only for fully-developed turbulent flows over a rough boundary,<br>
function of  $R_k/k$  but not  $Re$ . If *n* is truly a measure of the surface rough order that independent of either  $R_n/k$  or  $Re$ . Strickler (1923) proposed that<br>that independent of either  $R_n/k$  or  $Re$ . Strickler (1923) proposed that<br>(12)<br>aition of *n* with the flow depth, Eqs.(9d) and (10) can be combin extraction of  $R_s/k$  but not  $Re$ . If  $n$  is truly a measure of the surface roughness,<br>
a constant independent of either  $R_s/k$  or  $Re$ . Strickler (1923) proposed that<br>
0342 (12)<br>
(12)<br>
be variation of  $n$  with the flow depth *n* **a** *n n n n n n n n* **<b>***n k* is a function of  $R_h/k$  but not  $Re$ . If *n* is truly a measure of the surface roughness,<br>will be a constant independent of either  $R_h/k$  or  $Re$ . Strickler (1923) proposed that<br> $\frac{n}{k^{1/5}} = 0.0342$  (12)<br> $\frac{n}{k^{1/5}} = 0.03$ 

$$
\frac{n}{k^{1/6}} = 0.0342\tag{12}
$$

To investigate the variation of *n* with the flow depth, Eqs.(9d) and (10) can be combined to give e variation of *n* with the flow depth, Eqs.(9d) and (1<br>  $\frac{3g}{\sqrt{n}} \left( 2 \log \frac{12R_h}{k} \right)$ <br>  $\frac{m}{\sqrt{n}} \left( \frac{(R_h/k)^{1/6}}{2 \log(12R_h/k)} \right)$ 

$$
\frac{R_h^{1/6}}{n} = \frac{\sqrt{8g}}{C_m} \left( 2\log\frac{12R_h}{k} \right)
$$
 (13)

or

$$
\frac{n}{k^{1/6}} = \frac{C_m}{\sqrt{8g}} \frac{(R_h/k)^{1/6}}{2\log(12R_h/k)}
$$
(14)

Rouse (1946) plotted Eq.(13) to demonstrate the insensitivity of Manning's *n* to *R<sup>h</sup>* . This plot Steady Uniform Flow<br>
Rouse (1946) plotted Eq.(13) to demonstrate the insensitivity of Manning's *n* to  $R_h$ . This plot<br>
is shown in Fig. 2 ( $R_h/k$  versus  $R_h^{1/6}/n$ ) together with Stricker's formula and a straight line<br>
wi Steady Uniform Flow<br>3) to demonstrate the insensitivity of Manning's *n* to  $R_h$ . This plot<br>versus  $R_h^{1/6}/n$  ) together with Stricker's formula and a straight line<br>ough the point corresponding to the minimum value of  $n/k$ is shown in Fig. 2 ( $R_h / k$  versus  $R_h^{1/6} / n$ ) together with Stricker's formula and a straight line Steady Uniform Flow<br>Rouse (1946) plotted Eq.(13) to demonstrate the insensitivity of Manning's *n* to  $R_h$ . This plot<br>is shown in Fig. 2  $(R_h / k$  versus  $R_h^{1/6} / n$  logether with Stricker's formula and a straight line<br>with figure indicates that Eq.(13) coincides well with Strickler's formula, meaning that *n* is not sensitive to *R<sup>h</sup>* .





An alternative plot, perhaps more straightforward, to show the small variation of n with respect to  $R_h$  is given in Fig. 3 as done by Chow (1959, p.206). Figure 3 shows the change of  $n/k^{1/6}$ Eq. 17<br>
Eq. 17<br>
Eq. 17<br>
Eq. 17<br>
Figure 3. Variation of  $n/k^{1/6}$  with  $R_n/k$ <br>
An alternative plot, perhaps more straightforward, to show the small variation of n with respect<br>
to  $R_n$  is given in Fig. 3 as done by Chow (19 with  $R_h/k$ . In the figure, Eq.(14) is plotted with Strickler's formula. It can be seen that *n* changes little with *R<sup>h</sup>* . Note also that a thousandfold change in *k* results in a threefold change in *n*. This means that *k* is more sensitive than *n*. Therefore, it can be said if sands in the channel bed are uniform, Manning's *n* is constant along the channel, *f* changes a little and *C* changes significantly. Plotted also are similar relations for fully developed flow for hydraulically

smooth boundary (denoted by Eq.16) and for flow in the transition region (denoted by Eq.17). In such case, n is a function of  $R_h / k$  and for flow in the transition region (denoted by Eq.17).<br>In such case, n is a function of  $R_h / k$  and Reynolds number.<br>4. Conveyance In such case, n is a function of  $R_h/k$  and Reynolds number. Steady Uniform Flow<br> **Steady Uniform Flow**<br> **Condary (denoted by Eq.16) and for flow in the transition region (denoted by Eq.17).**<br> **Condary A Fig. 7 and Reynolds number.**<br> **Condary A Fig. 7 and Reynolds number.**<br> **Condar** boundary (denoted by Eq.16) and for flow in the transition region (denoted by Eq.17).<br>
rease, n is a function of  $R_h/k$  and Reynolds number.<br> **K** = *K* \left and Reynolds number.<br> **Example 8**<br> **K** =  $\frac{1}{\sqrt{S}}$  (15)<br> **K**

## **4. Conveyance**

When either the Chezy formula or Manning formula is used for uniform flow computation, the discharge becomes **Example 10**<br> **K** EV EXP formula or Manning formula is used for uniform flow computation, the<br> **K** EV EXP **K**<br> **K** =  $Q/\sqrt{S}$  (15)<br> **K** EQ /  $\sqrt{S}$  (16)<br> **K** =  $Q/\sqrt{S}$  (16)<br> **K** =  $CA\sqrt{R_s}$  (17)<br> **K** =  $CA\sqrt{R_s}$  (17)<br> fither the Chezy formula or Manning formula is used for uniform flow computation, the<br>
ge becomes<br>  $R = E\sqrt{S}$  (15)<br>
conveyance is<br>  $K = Q/\sqrt{S}$  (16)<br>
veyance is a measure of carrying capacity of the channel section. When Ch

$$
Q = K\sqrt{S} \tag{15}
$$

and the conveyance is

$$
K = Q / \sqrt{S} \tag{16}
$$

The conveyance is a measure of carrying capacity of the channel section. When Chezy formula is used, the conveyance is

$$
K = CA\sqrt{R_h} \tag{17}
$$

and when Manning formula is used,

$$
K = \frac{C_m}{n} A R_h^{2/3} \tag{18}
$$

## **5. Best Hydraulic Sections**

What is the best hydraulic cross section in designing the channel (not analyzing the channel)? Some channel cross sections are more efficient than others in that they provide more area for a given wetted parameter. Or the sections have the least wetted perimeter for a given cross sectional area.

(1) Rectangular Cross Section

With the help of Manning's equation, if  $Q$ , *n*, and *S* are known, the cross sectional area can be expressed as a function of perimeter *P* as





$$
A = cP^{2/5} \tag{19}
$$

$$
(P - 2h)h = cP^{2/5} \tag{20}
$$

Differentiating eq.(19) with respect to *y* results

$$
\left(\frac{dP}{dh} - 2\right)h + (P - 2h) = \frac{2}{5}cP^{-3/5}\frac{dP}{dh}
$$
\n(21)

Setting  $dP/dh = 0$  gives  $P = 4h$ , or

$$
b = 2h \tag{22}
$$

*b* =  $\frac{A}{2} = cP^{2/3}$  (19)<br> *h* =  $\frac{1}{2}$  fix flamed cross sections (19)<br> *h* is known. For a rectangular channel, the width (*B*) is  $P - 2h$ , so<br>  $(P - 2h)h = cP^{2/3}$  (20)<br> *h* intating eq.(19) with respect to *y* resul That is, the best rectangular hydraulic section has the depth which is one-half of the width. (Q) Obtain the best hydraulic section of a rectangular cross section with a free board *F* at both sides. (2) Trapezoidal Cross Section

For the trapezoidal section, similarly,

Steady Uniform Flow  
\n
$$
bh + mh^2 = (P - 2h\sqrt{1 + m^2})h + mh^2 = cP^{2/5}
$$
 (23)  
\nDifferentiating eq.(22) with respect to *h* yields  
\n $P = 4h\sqrt{1 + m^2} - 2mh$  (24)  
\nNow, for a constant water depth (*h*), *m* is to be sought. That is,  $dP/dm = 0$  leads to  
\n $m = 1/\sqrt{3}$  (25)  
\nTherefore, the best hydraulic section is one-half of a hexagon.  
\n(Q) Best hydraulic section in terms of sediment transport

Differentiating eq.(22) with respect to *h* yields

$$
P = 4h\sqrt{1+m^2} - 2mh\tag{24}
$$

$$
m = 1/\sqrt{3} \tag{25}
$$

Therefore, the best hydraulic section is one-half of a hexagon.

(Q) Best hydraulic section in terms of sediment transport

## **6. Flows in Composite Roughness Channel and Compound Channel**

Consider the channel section of composite roughness. Various methods for computing the equivalent roughness are available. A simple method proposed by Horton (1993) assumes that the velocities in the sub-sections are the same as the average velocity over the whole cross section. That is, the mean velocity in the *i*-th section is given by channel section of composite roughness. Variou<br>ughness are available. A simple method proposed b<br>in the sub-sections are the same as the average<br>is, the mean velocity in the *i*-th section is given by<br> $\left(\frac{A_i}{P_i}\right)^{2/3} S$ Free, the best hydraulic section is one-half of a hexagon.<br> **V** sin Composite Roughness Channel and Compound Channel<br> **Y** in Composite Roughness Channel and Compound Channel<br> **Property** the channel section of composite ro incomposite Roughness Channel and Compound Channel<br> *n* Composite Roughness Channel and Compound Channel<br> *n* Composite Roughness Channel and Compound Channel<br> *n* endmanel section of composite roughness. Various methods  $=1/\sqrt{3}$  (25)<br>the best hydraulic section is one-half of a hexagon.<br>ydraulic section in terms of sediment transport<br>**n** Composite Roughness Channel and Compound Channel<br>the channel section of composite roughness. Various *i*, the mean velocity in the *i*-th section is given by<br> *i*, the mean velocity in the *i*-th section is given by<br>  $\left(\frac{A_i}{P_i}\right)^{2/3} S^{1/2}$ <br> **i**  $= \sum_i P_i n_i^{3/2} \frac{V^{3/2}}{S^{3/4}}$ <br> *i* s formula, the total area is also gi It is, the mean velocity in the *i*-th section is given by<br>  $\frac{1}{n_i} \left(\frac{A_i}{P_i}\right)^{2/3} S^{1/2}$ <br>
ea is expressed as<br>  $\sum_{i} A_i = \sum_{i} P_i n_i^{3/2} \frac{V^{3/2}}{S^{3/4}}$ <br>
ing's formula, the total area is also given by<br>  $V^{2/3} P_{i} 2^{1$ **A A A A Properties Roughness Channel and Compound Channel**<br> *A* composite roughness. Various methods for computing the<br> *A* Lettroughness are available. A simple method proposed by Horton (1993) assumes that<br> **A** coities a constant water depth (*b*), *m* is to be sought. That is,  $dP/dm = 0$  leads to<br>  $s = 1/\sqrt{3}$ <br>
(25)<br>
e, the best hydraulic section is one-half of a hexagon.<br>
Hydraulic section in terms of sediment transport<br>
in Composite Ro *r* the channel section of composite roughness. Various methods for computing the nt roughness are available. A simple method proposed by Horton (1993) assumes that cities in the sub-sections are the same as the average v

$$
V_i = \frac{1}{n_i} \left(\frac{A_i}{P_i}\right)^{2/3} S^{1/2}
$$

The total area is expressed as

$$
A = \sum_{i} A_{i} = \sum_{i} P_{i} n_{i}^{3/2} \frac{V^{3/2}}{S^{3/4}}
$$

From Manning's formula, the total area is also given by

$$
A = \frac{V^{2/3} P n^{2/3}}{S^{3/4}}
$$

 $A_i = \sum_i P_i n_i^{3/2} \frac{V^{3/2}}{S^{3/4}}$ <br>
g's formula, the total area is also given by<br>  $2^{2/3} P n^{2/3} \frac{V^{3/2}}{S^{3/4}}$ <br>
elationships for the areas leads to the equivalent roughly Equating the relationships for the areas leads to the equivalent roughness coefficient such as

Steady Uniform Flow  
\n
$$
n = \frac{\left(\sum_{i} P_{i} n_{i}^{3/2}\right)^{2/3}}{P^{2/3}}
$$
\n(26)  
\nrly, for compound channel section, the mean velocity is given by  
\n
$$
V_{i} = \frac{1}{n_{i}} \left(\frac{A_{i}}{P_{i}}\right)^{2/3} S^{1/2}
$$
\n
$$
P_{i} = \frac{1}{n_{i}} \left(\frac{A_{i}}{P_{i}}\right)^{2/3} S^{1/2}
$$
\n
$$
Q = \sum_{i} A_{i} V_{i}
$$
\n(27)

Similarly, for compound channel section, the mean velocity is given by

 $\sum_{i=1}^{7} = \frac{1}{n} \left( \frac{A_i}{B} \right)^{2/3} S^{1/2}$  $A_i$   $\int_{0}^{1/2}$ 

Thus the total discharge is given by

$$
n = \frac{(\frac{\sum_{i} P_{i}v_{i}}{P^{2/3}})}{P^{2/3}}
$$
\n\narly, for compound channel section, the mean velocity is given by

\n
$$
V_{i} = \frac{1}{n_{i}} \left(\frac{A_{i}}{P_{i}}\right)^{2/3} S^{1/2}
$$
\nThe total discharge is given by

\n
$$
Q = \sum_{i} A_{i}V_{i}
$$
\n(27)



(a) channel with different roughnesses (b) compound channel section

It should be noted that composite roughness can be used in assessing the discharge when the velocity structure is relatively homogeneous over the entire section. Otherwise, mean velocity should be estimated at each section such as in a compound channel.

# **References**

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## **Problems**

1. Show that if sands in the channel bed is uniform, Manning's *n* is constant along the channel, Darcy Weisbach's *f* changes a little and Chezy's *C* changes significantly.

2. Obtain the best hydraulic section of a rectangular cross section with a free board *F* at both sides.

3. In general, the wide open-channel can safely defined as a rectangular channel whose width is greater than 10-15 times the depth of flow. That is,

$$
B/y = 10-15
$$

*B* w that if sands in the channel bed is uniform, Manning's *n* is constant along the channel,<br>*Weisbach's f* changes a little and Chezy's *C* changes significantly.<br>
A *B* with a free board *F* at both<br>
pain the best hy where  $B$  is the width and  $y$  is the flow depth. In the wide open-channel, the dynamics due to the circulations in the direction transverse to the main flow direction can be ignored. Consider the (rectangular-shaped) open-channel at the hydraulic laboratory in Yonsei University. The width of the channel is about 1 m, and the flow depth of  $y = 0.25$  m is going to be maintained. The side wall is made of glass ( $n = 0.01$ ) and the channel bottom is covered by the concrete block ( $n = 0.03$  assumed) to supply extra roughness. Can this channel be considered as a wide rectangular open-channel?

4. Derive the governing equation for long wave theory which can be applied to many problems in open-channel flows by averaging the following continuity and momentum equations: Steady Uniform Flow<br>tive the governing equation for long wave theory which can be applied to many problem-<br>channel flows by averaging the following continuity and momentum equations:<br> $\nabla \cdot \vec{V} = 0$ <br> $\frac{d\vec{V}}{dt} = -\frac{1}{\rho$ Steady Uniform Flow<br>
the governing equation for long wave theory which can be applied to many problems<br>
annel flows by averaging the following continuity and momentum equations:<br>  $\vec{r} = 0$ <br>  $= -\frac{1}{\rho} \nabla p^*$ <br>  $= p + \gamma z$ . Steady Uniform Flow<br>
4. Derive the governing equation for long wave theory which can be applied to many problems<br>
in open-channel flows by averaging the following continuity and momentum equations:<br>  $\nabla \cdot \vec{r} = 0$ <br>  $\frac{d$ 

$$
\nabla \cdot \vec{V} = 0
$$

$$
\frac{d\vec{V}}{dt} = -\frac{1}{\rho} \nabla p^*
$$